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DEMONSTRATING NON-LINEAR RLC CIRCUIT EQUATION BY INVOLVING FRACTIONAL ADOMIAN DECOMPOSITION METHOD BY CAPUTO DEFINITION

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Abstract. This research displays nonlinear RLC circuit equation applying the fractional Adomian Decomposition Method (ADM) to investigate an approximate analytical solution. Where the fractional derivative described here as in the Caputo definition. The result behavior obtained by ADM is displayed graphically and numerically where it indicates a great consist compared with those obtained by other methods.

Keywords: RLC circuit, fractional Non-Linear differential equation, Caputo definition, Adomian decomposition method.

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1 Introduction

Many physical, mechanical, chemical, biological and other scientific fields, problems or phenomenons (Asif et al., 2018; Can et al., 2020; Yavuz et al., 2021; Akour et al., 2023, 2022; Jafari et al., 2023; Ma et al., 2016; Guzman et al., 2021) are stated in mathematical differential equation models. These models are applied to investigate qualitative and quantitative scientific characteristics that perform the significant problems or phenomenons. Nonlinear phenomenons or problems require additional strategies and more advanced techniques to provide a specified or approximate solution for these models. The significant method for solving nonlinear equations known as fractional nonlinear differential equations (FNDE) which spread over a wide application in science and engineering. Recently, a lot of effort has been concentrated into developing accurate and consistent numerical and analytical methods to satisfy fractional differential equations (FDE) for its significant importance. A FDE solution proves a significant development, that there is generally no approach that gives a specific solution to a FDE. FDE can be used

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to display an enormous particular physical case topics of nonlinear equations known as FNDE which has various applications in science and engineering. The Fractional Variation Iteration Method (Wu, 2023), Homotopy Analysis Method (Hamarsheh et al., 2016), and Fractional Differential Transform Method (Günerhan & Çelik, 2020), are just some techniques to solve these equations. RLC circuit as an example a nonlinear electric oscillator where many physical and electric engineers researcher try to provide an accurate model to predict its physical and electrical properties and develop its services. Resonant electrical circuits are familiar widely used in electronic devices. Where Resistive, inductive, and capacitive unit construct the basis of nonlinearity in resonant electrical systems. But to understand and predict nonlinear electrical properties from a theoretical and practical point of view required solving the nonlinear RLC circuit equation. Previous study like Langragion method (Kemle & Beyer, 2020) describe successfully nonlinear RLC circuits but for a nonlinear capacitor. (HPTM) is applied to provide a good approximation to solve the nonlinear RLC circuit equation for nonlinear inductance (Thunibat et al., 2021). The main purpose of this work is to provide a significant description for the fractional nonlinear RLC circuit equation in case of nonlinear inductance by the applying the Adomian Decomposition Method (ADM). Otherwise, in this case, ADM is for the first time selected to displays nonlinear RLC circuit equation with nonlinear inductance and thus would provide another computable and supporting description comparing with other method. The (ADM) is constructed from accumulated branches of the solution of nonlinear equation in series of function that derived from a polynomial developed by power series expansion of analytic function (González-Gaxiola, 2017). This work is arranged as follows; Section 2, which provides some essential concepts and outcomes related to the fractional derivative; Section 3, which exhibit the methodology of the (ADM) for solving (FNDE); section 4, where (ADM) is engaged to involve an approximate analytical solution to the fractional nonlinear RLC circuit equation; Section 5, where the result is expressed numerically evaluated by (ADM) and then compared with those obtained by other methods. Also it displays graphically the behaviour of solution and the effect for different values of fractional order; finally, section 6 provides the conclusions.

2 Fractional Calculus

Fractional calculus was established when the famous mathematician L'Hopial, asked his counterpart Leibniz to provide an explanation for exchanging the integer order n of the derivative (dn/dxn) to be a fragment such as n=1/2. Before a few decades, this topic was considered a purely mathematical topic and was only of mathematician's interest. Euler, Fourier, Abel, Liouville, Riemann, Hadamard, among others, have studied these new fractional operators, by presenting new definitions and studying their most important properties. However, in the past decades, this subject has demonstrated its applicability in many different natural science phenomenons and problems as we mentioned earlier. Numerous definitions for fractional derivative and integral are developed to this fruitful field. Recently, the most often used of fractional derivatives are, the Grünwald–Letnikov derivative, the Riemann–Lioule derivative, and the Caputo derivative. In this work, we use the Caputo fractional (CF) derivative. CF derivative definition:

$$D^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^x \frac{f^n(\tau)}{(t-\tau)^{\alpha+1-n}}\, d\tau, \tag{1}$$

where $n = 1, 2, 3, \ldots, n \in N$ and $n - 1 < \alpha \leq n$. The main CF derivative's characters (Samko et al., 2017):

$$[D^{\alpha} f(t)] = s^{\alpha} F(s) - \sum_{k=0}^{n-1} s^{\alpha-k-1} f^{(k)}(0), \tag{2}$$

where $n \in N$ and $-1 < \alpha \leq n$. The main CF derivative's characters (Samko et al., 2017):
Representation: The fractional integral is defined as:

$$J^{\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t f(\tau)(t-\tau)^{\alpha-1}\, d\tau. \tag{3}$$

Insert

$$f(t) = D^n f(t). \tag{4}$$

In equation (3), and replace

$$\alpha = n - \alpha. \tag{5}$$

Then, equation (3) become

$$\begin{aligned} J^{(n-\alpha)} D^n f(t) &= \frac{1}{\Gamma(n-\alpha)} \int_0^{\infty} D^n f(\tau)(t-\tau)^{n-\alpha-1}\, d\tau \\ &= \frac{1}{\Gamma(n-\alpha)} \int_0^{\infty} f^{(n)}(\tau)(t-\tau)^{n-\alpha-1}\, d\tau \\ &= D^{\alpha} f(t) \end{aligned} \tag{6}$$

So,

$$D^{\alpha} f(t) = J^{(n-\alpha)} D^n f(t). \tag{7}$$

This implies that CF operator is equal to $(n-\alpha)$ order integration after n^{th} order differentiation.
Interpolation: let $n \in N$ and $n-1 < \alpha \leq n$, and f(t) be such that $D^{\alpha} f(t)$ exists, then:

$$\lim_{\alpha \to n} D^{\alpha} f(t) = f^n(t), \tag{8}$$

$$\lim_{\alpha \to n-1} D^{\alpha} f(t) = f^{(n-1)}(t) - f^{n-1}(0). \tag{9}$$

Linearity: The functions f(t) and g(t) will be such that both $D^{\alpha} f(t)$ and $D^{\alpha} g(t)$ exist.
The CF derivative is a linear operator

$$D^{\alpha}(\lambda f(t) + g(t)) = \lambda D^{\alpha} f(t) + D^{\alpha} g(t). \tag{10}$$

Non-commutation: In general

$$D^{\alpha} D^m f(t) = D^{\alpha+m} f(t) \neq D^m D^{\alpha} f(t) \tag{11}$$

$$D^{\alpha} f(t) = D^{\beta} D^{n-1} f(t). \tag{12}$$

Suppose that $n-1 < \alpha \leq n$, $\beta = \alpha - (n-1)$, $0 < \beta < 1$, $n < N$, $\alpha, \beta \in R$, and the function $f(t)$ is such that $D^{\alpha} f(t)$ exists, then

$$D^{\alpha} f(t) = D^{\beta} D^{n-1} f(t).$$

3 Fractional Adomian Decomposition Method

This technique was developed by George Adomian in 1980, by dissociation the NDE in a series of functions. Next, each function developed from a polynomial involved by extension a power series of an analytical function. Thus, the linear and nonlinear parts are determined individually. Then, Laplace transforms and its inverse operate to the linear part function to obtain a solution pretended by the inverse operator. Thus the Adomian Polynomials will be hold and the nonlinear part can hold by these polynomials. Finally, one can investigate a recursion relation considering the proper initial condition. This method is so easy in its common form and spread widely in solving NDE and FNDE.

The interested FNDE with it's initial conditions is in the form

$$D^{\alpha}u(t) + Ru(t) + Nu(t) = g, \quad 1 \le \alpha < 2, \tag{13}$$

$$u(0) = 0, \quad \dot{u}(0) = 0, \tag{14}$$

where D^{α} is an operator determine the order of the fractional derivative expressed in the Caputo sense, R is the linear part term, N denotes to the nonlinear differential operator, and g is the source term.

Developing Laplace transform for equation (13).

$$\mathcal{L}[D^{\alpha}u(t)] + \mathcal{L}[Ru(t)] + \mathcal{L}[Nu(t)] = \mathcal{L}[g]. \tag{15}$$

We catch

$$\begin{aligned} \mathcal{L}[D^{\alpha}u(t)] &= s^{\alpha}\mathcal{L}[u(t)] - s^{\alpha-1}u(0) - s^{\alpha-2}\dot{u}(0) \\ &= s^{\alpha}\mathcal{L}[u(t)]. \end{aligned} \tag{16}$$

So, equation (15) becomes

$$\mathcal{L}[u(t)] = \frac{1}{s^{\alpha}}\mathcal{L}[g] - \frac{1}{s^{\alpha}}\mathcal{L}[Ru(t)] - \frac{1}{s^{\alpha}}\mathcal{L}[Nu(t)]. \tag{17}$$

Now, providing Inverse Laplace transform in above equation:

$$\mathcal{L}^{-1}[\mathcal{L}[u(t)]] = \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}[g]\right] - \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}[Ru(t)]\right] - \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}[Nu(t)]\right] \tag{18}$$

$$u(t) = \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}[g]\right] - \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}[Ru(t)]\right] - \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}[Nu(t)]\right]. \tag{19}$$

Now substitute the linear term $u(t)$ by an infinite series.

$$u(t) = \sum_{n=0}^{\infty} u_n(t). \tag{20}$$

The Non-linear term $Nu(t)$ is determined by an infinite series

$$Nu(t) = \sum_{n=0}^{\infty} A_n, \tag{21}$$

where the Adomian polynomials A_n of $(u_1, u_2, \ldots, u_n)$ can be expressed by

$$A_n(u_1, u_2, \ldots, u_n) = \frac{1}{n!}\left(\frac{d^n}{d\lambda^n}\right)\left[N\left(\sum_{i=0}^{\infty}\lambda^i u_i\right)\right], \quad n = 0, 1, 2, 3, \ldots. \tag{22}$$

Now, the equation (19)becomes

$$\sum_{n=0}^{\infty} u_n(t) = \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}[g]\right] - \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}\left[\sum_{n=0}^{\infty} u_n(t)\right]\right] - \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}\left[\sum_{n=0}^{\infty} A_n\right]\right]. \tag{23}$$

Then from matching the both sides of equation (23), we get

$$u_0(t) = \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}[g]\right], \tag{24}$$

$$u_1(t) = -\mathcal{L}^{-1}\left[\frac{1}{s^\alpha}\mathcal{L}[u_0(t)]\right] - \mathcal{L}^{-1}\left[\frac{1}{s^\alpha}\mathcal{L}[A_0]\right], \tag{25}$$

$$u_1(t) = -\mathcal{L}^{-1}\left[\frac{1}{s^\alpha}\mathcal{L}[u_0(t)]\right] - \mathcal{L}^{-1}\left[\frac{1}{s^\alpha}\mathcal{L}[A_0]\right], \tag{26}$$

$$u_2(t) = -\mathcal{L}^{-1}\left[\frac{1}{s^\alpha}\mathcal{L}[u_1(t)]\right] - \mathcal{L}^{-1}\left[\frac{1}{s^\alpha}\mathcal{L}[A_1]\right], \tag{27}$$

$$u_3(t) = -\mathcal{L}^{-1}\left[\frac{1}{s^\alpha}\mathcal{L}[u_2(t)]\right] - \mathcal{L}^{-1}\left[\frac{1}{s^\alpha}\mathcal{L}[A_2]\right]. \tag{28}$$

Generally, the term u_{n+1} is given by

$$u_{n+1}(t) = -\mathcal{L}^{-1}\left[\frac{1}{s^\alpha}\mathcal{L}[u_n(t)]\right] - \mathcal{L}^{-1}\left[\frac{1}{s^\alpha}\mathcal{L}[A_n]\right]. \tag{29}$$

So, finally we can written the approximation of solution as

$$u(t) = \sum_{n=0}^{\infty} u_n(t). \tag{30}$$

4 Solving Non-Linear RLC Circuit Equation By Fractional Adomian Decomposition Method

The nonlinear RLC circuit equation has been shown and derived in a previous work (Thunibat et al., 2021) in detail and we have obtained a nonlinear second order differential equation. In this section we consider the Fractional Non-Linear RLC circuit equation as

$$D^\alpha \varphi(t) + \beta\varphi(t) + \gamma\varphi^3(t) = V\cos\omega t, \tag{31}$$

where

$$1 < \alpha \leq 2 \quad \text{and} \quad t \geq 0. \tag{32}$$

And, the initial conditions

$$\varphi(0) = 0 \quad \text{and} \quad \dot{\varphi}(0) = 0. \tag{33}$$

Developing Laplace transform in equation(31)

$$\mathcal{L}[D^\alpha \varphi(t)] + \beta\mathcal{L}[\varphi(t)] + \gamma\mathcal{L}[\varphi^3(t)] = V\mathcal{L}[\cos(\omega t)]. \tag{34}$$

We catch

$$\mathcal{L}[D^\alpha \varphi(t)] = s^\alpha \mathcal{L}[\varphi(t)]. \tag{35}$$

So, equation (34) becomes

$$\mathcal{L}[\varphi(t)] = \frac{V}{s^\alpha}\left(\frac{s}{s^2+\omega^2}\right) - \frac{\beta}{s^\alpha}\mathcal{L}[\varphi(t)] - \frac{\gamma}{s^\alpha}\mathcal{L}[\varphi^3(t)]. \tag{36}$$

Now, providing Inverse Laplace transform in above equation

$$\varphi(t) = V\mathcal{L}^{-1}\left[\frac{1}{s^\alpha}\mathcal{L}[\cos\omega t]\right] - \beta\mathcal{L}^{-1}\left[\frac{1}{s^\alpha}\mathcal{L}[\varphi(t)]\right] - \gamma\mathcal{L}^{-1}\left[\frac{1}{s^\alpha}\mathcal{L}[\varphi^3(t)]\right]. \tag{37}$$

Now exchanging the linear terms $\varphi(t)$ by an infinite series.

$$\varphi(t) = \sum_{n=0}^{\infty} \varphi_n(t). \tag{38}$$

While the Non-linear one $\varphi(t)^3$ is defined by an infinite series

$$\varphi^3(t) = \sum_{n=0}^{\infty} A_n(\varphi_0, \varphi_1, \ldots). \tag{39}$$

From equation (22) can be calculated, the first fourth Adomian polynomial of An as

$$A_0 = \varphi_0^3. \tag{40}$$

$$A_1 = 3\varphi_0^2\varphi_1, \tag{41}$$

$$A_2 = 3\varphi_0^2\varphi_2 + 3\varphi_0\varphi_1^2, \tag{42}$$

$$A_3 = 3\varphi_0^2\varphi_2 + 6\varphi_0\varphi_1\varphi_2 + \varphi_1^3, \tag{43}$$

Now, the equation (37) becomes

$$\sum_{n=0}^{\infty} \varphi_n(t) = V\mathcal{L}^{-1}\left[\frac{1}{s^\alpha}\mathcal{L}[\cos\omega t]\right] - \beta\mathcal{L}^{-1}\left[\frac{1}{s^\alpha}\mathcal{L}\left[\sum_{n=0}^{\infty}\varphi_n(t)\right]\right] - \gamma\mathcal{L}^{-1}\left[\frac{1}{s^\alpha}\mathcal{L}\left[\sum_{n=0}^{\infty}A_n(t)\right]\right]. \tag{44}$$

Then from matching the both sides of equation (44), we get

$$\varphi_0(t) = V\mathcal{L}^{-1}\left[\frac{1}{s^\alpha}\mathcal{L}[\cos(\omega t)]\right], \tag{45}$$

$$\varphi_1(t) = -\beta\mathcal{L}^{-1}\left[\frac{1}{s^\alpha}\mathcal{L}[\varphi_0(t)]\right] - \gamma\mathcal{L}^{-1}\left[\frac{1}{s^\alpha}\mathcal{L}[A_0]\right], \tag{46}$$

$$\varphi_2(t) = -\beta\mathcal{L}^{-1}\left[\frac{1}{s^\alpha}\mathcal{L}[\varphi_1(t)]\right] - \gamma\mathcal{L}^{-1}\left[\frac{1}{s^\alpha}\mathcal{L}[A_1]\right], \tag{47}$$

$$\varphi_3(t) = -\beta\mathcal{L}^{-1}\left[\frac{1}{s^\alpha}\mathcal{L}[\varphi_2(t)]\right] - \gamma\mathcal{L}^{-1}\left[\frac{1}{s^\alpha}\mathcal{L}[A_2]\right], \tag{48}$$

Generally, the term φ_{n+1} is given by

$$\varphi_{n+1}(t) = -\beta\mathcal{L}^{-1}\left[\frac{1}{s^\alpha}\mathcal{L}[\varphi_n(t)]\right] - \gamma\mathcal{L}^{-1}\left[\frac{1}{s^\alpha}\mathcal{L}[A_n]\right], \tag{49}$$

Now, in equation (45)

$$\cos\omega t = \sum_{n=0}^{\infty}(-1)^n\frac{(\omega t)^{2n}}{2n!} \tag{50}$$

and

$$\begin{aligned}\mathcal{L}[\cos\omega t] &= \mathcal{L}\left[\sum_{n=0}^{\infty}(-1)^n\frac{(\omega t)^{2n}}{2n!}\right]\\ &= \sum_{n=0}^{\infty}\frac{(-1)^n(\omega)^{2n}}{2n!}\mathcal{L}[(t)^{2n}]\\ &= \sum_{n=0}^{\infty}\frac{(-1)^n(\omega)^{2n}}{2n!}\left(\frac{2n!}{s^{2n+1}}\right)\\ &= \sum_{n=0}^{\infty}(-1)^n(\omega)^{2n}\left(\frac{1}{s^{2n+1}}\right)\end{aligned} \tag{51}$$

So,

$$\begin{aligned}
\mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}[\cos\omega t]\right] &= \mathcal{L}^{-1}\left[\sum_{n=0}^{\infty}(-1)^n(\omega)^{2n}\left(\frac{1}{s^{2n+\alpha+1}}\right)\right] \\
&= \sum_{n=0}^{\infty}(-1)^n(\omega)^{2n}\mathcal{L}^{-1}\left[\frac{1}{s^{2n+\alpha+1}}\right] \\
&= \sum_{n=0}^{\infty}(-1)^n(\omega)^{2n}\left(\frac{t^{2n+\alpha}}{(2n+\alpha)!}\right) \\
&= \frac{1}{\omega^{\alpha}}\sum_{n=0}^{\infty}(-1)^n\frac{(\omega t)^{2n+\alpha}}{(2n+\alpha)!}
\end{aligned} \tag{52}$$

So, the solution of equation (45) is

$$\varphi_0(t) = \frac{V}{\omega^{\alpha}}\sum_{n=0}^{\infty}(-1)^n\frac{(\omega t)^{2n+\alpha}}{(2n+\alpha)!}. \tag{53}$$

Now, in equation (46)

$$\begin{aligned}
\mathcal{L}[\varphi_0(t)] &= \mathcal{L}\left[\mu\sum_{n=0}^{\infty}(-1)^n\frac{(\omega t)^{2n+\alpha}}{(2n+\alpha)!}\right] \\
&= \mu\sum_{n=0}^{\infty}\frac{(-1)^n(\omega)^{2n+\alpha}}{(2n+\alpha)!}\mathcal{L}[(t)^{2n+\alpha}] \\
&= \mu\sum_{n=0}^{\infty}\frac{(-1)^n(\omega)^{2n+\alpha}}{(2n+\alpha)!}\left(\frac{(2n+\alpha)!}{s^{2n+\alpha+1}}\right) \\
&= \mu\sum_{n=0}^{\infty}(-1)^n(\omega)^{2n+\alpha}\left(\frac{1}{s^{2n+\alpha+1}}\right)
\end{aligned} \tag{54}$$

and

$$\begin{aligned}
\mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}[\varphi_0(t)]\right] &= \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\left(\mu\sum_{n=0}^{\infty}(-1)^n(\omega)^{2n+\alpha}\left(\frac{1}{s^{2n+\alpha+1}}\right)\right)\right] \\
&= \mu\sum_{n=0}^{\infty}(-1)^n(\omega)^{2n+\alpha}\mathcal{L}^{-1}\left[\frac{1}{s^{2n+2\alpha+1}}\right] \\
&= \mu\sum_{n=0}^{\infty}(-1)^n(\omega)^{2n+\alpha}\left(\frac{t^{2n+2\alpha}}{(2n+2\alpha)!}\right) \\
&= \frac{\mu}{\omega^{\alpha}}\sum_{n=0}^{\infty}(-1)^n\left(\frac{(\omega t)^{2n+2\alpha}}{(2n+2\alpha)!}\right)
\end{aligned} \tag{55}$$

So, the first term of equation (46)

$$-\beta\mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}[\varphi_0(t)]\right] = -\frac{\beta\mu}{\omega^{\alpha}}\sum_{n=0}^{\infty}(-1)^n\left(\frac{(\omega t)^{2n+2\alpha}}{(2n+2\alpha)!}\right). \tag{56}$$

Now,

$$\begin{aligned}
A_0 &= \varphi_0^3(t) \\
&= \mu^3\sum_{n=0}^{\infty}(-1)^{3n}\frac{(\omega t)^{6n+3\alpha}}{((2n+\alpha)!)^3}
\end{aligned} \tag{57}$$

and

$$\begin{aligned}\mathcal{L}[A_0] &= \mu^3 \sum_{n=0}^{\infty} \frac{(-1)^{3n}\omega^{6n+3\alpha}}{((2n+\alpha)!)^3}\mathcal{L}[(t)^{6n+3\alpha}] \\ &= \mu^3 \sum_{n=0}^{\infty} \frac{(-1)^{3n}\omega^{6n+3\alpha}}{((2n+\alpha)!)^3}\left(\frac{(6n+3\alpha)!}{s^{6n+3\alpha+1}}\right).\end{aligned} \tag{58}$$

So, the second term of equation (46) becomes

$$\begin{aligned}-\gamma\mathcal{L}^{-1}\left[\frac{1}{s^\alpha}\mathcal{L}[A_0]\right] &= -\gamma\mu^3 \sum_{n=0}^{\infty} \frac{(-1)^{3n}\omega^{6n+3\alpha}}{((2n+\alpha)!)^3}((6n+3\alpha)!)\mathcal{L}^{-1}\left[\frac{1}{s^{6n+4\alpha+1}}\right] \\ &= -\frac{\gamma\mu^3}{\omega^\alpha} \sum_{n=0}^{\infty} \frac{(-1)^{3n}((6n+3\alpha)!)}{((2n+\alpha)!)^3}\left(\frac{(\omega t)^{6n+4\alpha}}{(6n+4\alpha)!}\right).\end{aligned} \tag{59}$$

The solution of equation (46) becomes

$$\varphi_1(t) = -\frac{\beta\mu}{\omega^\alpha}\sum_{n=0}^{\infty}(-1)^n\left(\frac{(\omega t)^{2n+2\alpha}}{(2n+2\alpha)!}\right) - \frac{\gamma\mu^3}{\omega^\alpha}\sum_{n=0}^{\infty}\frac{(-1)^{3n}((6n+3\alpha)!)}{((2n+\alpha)!)^3}\left(\frac{(\omega t)^{6n+4\alpha}}{(6n+4\alpha)!}\right). \tag{60}$$

In equation (47)

$$\begin{aligned}\mathcal{L}[\varphi_1(t)] &= -\frac{\beta\mu}{\omega^\alpha}\sum_{n=0}^{\infty}(-1)^n\mathcal{L}\left[\frac{(\omega t)^{2n+2\alpha}}{(2n+2\alpha)!}\right] - \frac{\gamma\mu^3}{\omega^\alpha}\sum_{n=0}^{\infty}\frac{(-1)^{3n}((6n+3\alpha)!)}{((2n+\alpha)!)^3}\mathcal{L}\left[\frac{(\omega t)^{6n+4\alpha}}{(6n+4\alpha)!}\right] \\ &= -\frac{\beta\mu}{\omega^\alpha}\sum_{n=0}^{\infty}(-1)^n(\omega)^{2n+2\alpha}\left(\frac{1}{s^{2n+2\alpha+1}}\right) \\ &- \frac{\gamma\mu^3}{\omega^\alpha}\sum_{n=0}^{\infty}\frac{(-1)^{3n}((6n+3\alpha)!)}{((2n+\alpha)!)^3}(\omega)^{6n+4\alpha}\left(\frac{1}{s^{6n+4\alpha+1}}\right).\end{aligned} \tag{61}$$

and

$$\begin{aligned}\mathcal{L}^{-1}\left[\frac{1}{s^\alpha}\mathcal{L}[\varphi_1(t)]\right] &= -\frac{\beta\mu}{\omega^\alpha}\sum_{n=0}^{\infty}(-1)^n(\omega)^{2n+2\alpha}\mathcal{L}^{-1}\left[\frac{1}{s^{2n+3\alpha+1}}\right] \\ &- \frac{\gamma\mu^3}{\omega^\alpha}\sum_{n=0}^{\infty}\frac{(-1)^{3n}((6n+3\alpha)!)}{((2n+\alpha)!)^3}(\omega)^{6n+4\alpha}\mathcal{L}^{-1}\left[\frac{1}{s^{6n+5\alpha+1}}\right] \\ &= -\frac{\beta\mu}{\omega^\alpha}\sum_{n=0}^{\infty}(-1)^n(\omega)^{2n+2\alpha}\left(\frac{t^{2n+3\alpha}}{(2n+3\alpha)!}\right) \\ &- \frac{\gamma\mu^3}{\omega^\alpha}\sum_{n=0}^{\infty}\frac{(-1)^{3n}((6n+3\alpha)!)}{((2n+\alpha)!)^3}(\omega)^{6n+4\alpha}\left(\frac{t^{6n+5\alpha}}{(6n+5\alpha)!}\right).\end{aligned} \tag{62}$$

So, the first term of equation (47) becomes

$$\begin{aligned}&\mathcal{L}^{-1}\left[\frac{1}{s^\alpha}\mathcal{L}[\varphi_1(t)]\right] \\ &\qquad = \frac{\beta^2\mu}{\omega^{2\alpha}}\sum_{n=0}^{\infty}(-1)^n\frac{(\omega t)^{2n+3\alpha}}{(2n+3\alpha)!} \\ &\qquad + \frac{\beta\gamma\mu^3}{\omega^{2\alpha}}\sum_{n=0}^{\infty}\frac{(-1)^{3n}((6n+3\alpha)!)}{((2n+\alpha)!)^3}\left(\frac{(\omega t)^{6n+5\alpha}}{(6n+5\alpha)!}\right).\end{aligned} \tag{63}$$

Now,

$$\begin{aligned}
A_1 &= 3\varphi_0^2(t)\varphi_1(t) \\
&= 3\left(\mu\sum_{n=0}^{\infty}(-1)^n\frac{(\omega t)^{2n+\alpha}}{(2n+\alpha)!}\right)^2\left(-\frac{\beta\mu}{\omega^\alpha}\sum_{n=0}^{\infty}(-1)^n\left(\frac{(\omega t)^{2n+2\alpha}}{(2n+2\alpha)!}\right)\right. \\
&\quad\left.-\frac{\gamma\mu^3}{\omega^\alpha}\sum_{n=0}^{\infty}\frac{(-1)^{3n}((6n+3\alpha)!)}{((2n+\alpha)!)^3}\left(\frac{(\omega t)^{6n+4\alpha}}{(6n+4\alpha)!}\right)\right) \\
&= -\frac{3\mu^3\beta}{\omega^\alpha}\sum_{n=0}^{\infty}\frac{(-1)^{3n}(\omega)^{6n+4\alpha}}{((2n+\alpha)!)^2((2n+2\alpha)!)}t^{6n+4\alpha} \\
&\quad-\frac{3\gamma\mu^5}{\omega^\alpha}\sum_{n=0}^{\infty}\frac{(-1)^{5n}((6n+3\alpha)!)(\omega)^{10n+6\alpha}}{((2n+\alpha)!)^5((6n+4\alpha)!)}(t)^{10n+6\alpha}
\end{aligned} \tag{64}$$

and

$$\begin{aligned}
\mathcal{L}[A_1] &= -\frac{3\mu^3\beta}{\omega^\alpha}\sum_{n=0}^{\infty}\frac{(-1)^{3n}(\omega)^{6n+4\alpha}}{((2n+\alpha)!)^2((2n+2\alpha)!)}\mathcal{L}[(t)^{6n+4\alpha}] \\
&\quad-\frac{3\gamma\mu^5}{\omega^\alpha}\sum_{n=0}^{\infty}\frac{(-1)^{5n}((6n+3\alpha)!)(\omega)^{10n+6\alpha}}{((2n+\alpha)!)^5((6n+4\alpha)!)}\mathcal{L}[(t)^{10n+6\alpha}] \\
&= -\frac{3\mu^3\beta}{\omega^\alpha}\sum_{n=0}^{\infty}\frac{(-1)^{3n}(\omega)^{6n+4\alpha}}{((2n+\alpha)!)^2((2n+2\alpha)!)}\left(\frac{(6n+4\alpha)!}{s^{6n+4\alpha+1}}\right) \\
&\quad-\frac{3\gamma\mu^5}{\omega^\alpha}\sum_{n=0}^{\infty}\frac{(-1)^{5n}((6n+3\alpha)!)(\omega)^{10n+6\alpha}}{((2n+\alpha)!)^5((6n+4\alpha)!)}\left(\frac{(10n+6\alpha)!}{s^{10n+6\alpha+1}}\right).
\end{aligned} \tag{65}$$

So, the second term of equation (47) becomes

$$\begin{aligned}
&-\gamma\mathcal{L}^{-1}\left[\frac{1}{s^\alpha}\mathcal{L}[A_1]\right] \\
&= \frac{3\mu^3\beta\gamma}{\omega^\alpha}\sum_{n=0}^{\infty}\frac{(-1)^{3n}((6n+4\alpha)!)(\omega)^{6n+4\alpha}}{((2n+\alpha)!)^2((2n+2\alpha)!)}\mathcal{L}^{-1}\left[\frac{1}{s^{6n+5\alpha+1}}\right] \\
&\quad+\frac{3\gamma^2\mu^5}{\omega^\alpha}\sum_{n=0}^{\infty}\frac{(-1)^{5n}((6n+3\alpha)!)((10n+6\alpha)!)(\omega)^{10n+6\alpha}}{((2n+\alpha)!)^5((6n+4\alpha)!)}\mathcal{L}^{-1}\left[\frac{1}{s^{10n+7\alpha+1}}\right] \\
&= \frac{3\mu^3\beta\gamma}{\omega^\alpha}\sum_{n=0}^{\infty}\frac{(-1)^{3n}((6n+4\alpha)!)(\omega)^{6n+4\alpha}}{((2n+\alpha)!)^2((2n+2\alpha)!)}\left(\frac{t^{6n+5\alpha}}{(6n+5\alpha)!}\right) \\
&\quad+\frac{3\gamma^2\mu^5}{\omega^\alpha}\sum_{n=0}^{\infty}\frac{(-1)^{5n}((6n+3\alpha)!)((10n+6\alpha)!)(\omega)^{10n+6\alpha}}{((2n+\alpha)!)^5((6n+4\alpha)!)}\left(\frac{t^{10n+7\alpha}}{(10n+7\alpha)!}\right).
\end{aligned} \tag{66}$$

So, the solution of equation (47) becomes

$$\begin{aligned}
\varphi_2(t) &= \frac{(\beta^2\mu)}{\omega^{2\alpha}}\sum_{n=0}^{\infty}(-1)^n\left(\frac{(\omega t)^{2n+3\alpha}}{(2n+3\alpha)!}\right) \\
&\quad+\frac{\beta\gamma\mu^3}{\omega^{2\alpha}}\sum_{n=0}^{\infty}(-1)^{3n}\left(\frac{((6n+3\alpha)!)}{((2n+\alpha)!)^3}+\frac{3((6n+4\alpha)!)}{((2n+\alpha)!)^2((2n+2\alpha)!)}\right)\left(\frac{(\omega t)^{6n+5\alpha}}{(6n+5\alpha)!}\right) \\
&\quad+\frac{3\gamma^2\mu^5}{\omega^{2\alpha}}\sum_{n=0}^{\infty}\frac{(-1)^{5n}((6n+3\alpha)!)((10n+6\alpha)!)}{((2n+\alpha)!)^5((6n+4\alpha)!)}\left(\frac{(\omega t)^{10n+7\alpha}}{(10n+7\alpha)!}\right).
\end{aligned} \tag{67}$$

And the solution of equation (48) becomes

$$
\begin{aligned}
\varphi_3(t) = &-\frac{\beta^3\mu}{\omega^{3\alpha}}\sum_{n=0}^{\infty}(-1)^n\left(\frac{(\omega t)^{2n+4\alpha}}{(2n+4\alpha)!}\right)\\
&-\frac{\beta^2\gamma\mu^3}{\omega^{3\alpha}}\sum_{n=0}^{\infty}(-1)^{3n}\left(\frac{((6n+3\alpha)!)}{((2n+\alpha)!)^3}+\frac{3((6n+4\alpha)!)}{((2n+\alpha)!)^2(2n+2\alpha)!}\right.\\
&\left.+\frac{3((6n+5\alpha)!)}{((2n+\alpha)!)^2((2n+3\alpha)!)}+\frac{3((6n+5\alpha)!)}{((2n+\alpha)!)((2n+2\alpha)!)^2}\right)\left(\frac{(\omega t)^{6n+6\alpha}}{(6n+6\alpha)!}\right)\\
&-\frac{3\beta\gamma^2\mu^5}{\omega^{3\alpha}}\sum_{n=0}^{\infty}(-1)^{5n}\left(\frac{(((6n+3\alpha)!)(10n+6\alpha)!)}{((2n+\alpha)!)^5((6n+4\alpha)!)}\right.\\
&+\frac{((6n+3\alpha)!)((10n+7\alpha)!)}{((2n+\alpha)!)^2((6n+5\alpha)!)((2n+\alpha)!)^3}\\
&+\frac{3((6n+4\alpha)!)((10n+7\alpha)!)}{((2n+\alpha)!)^2(((6n+5\alpha)!)((2n+\alpha)!)^2(2n+2\alpha)!)}\\
&\left.+\frac{2((6n+3\alpha)!)((10n+7\alpha)!)}{((2n+\alpha)!)^2((6n+5\alpha)(2n+\alpha)!)((2n+\alpha)!)^2(2n+2\alpha)!}\right)\left(\frac{(\omega t)^{10n+8\alpha}}{(10n+8\alpha)!}\right)\\
&-\frac{9\gamma^3\mu^7}{\omega^{3\alpha}}\sum_{n=0}^{\infty}\frac{(-1)^{7n}((6n+3\alpha)!(10n+6\alpha)!)}{((2n+\alpha)!)((6n+4\alpha)!)((2n+\alpha)!)^6}\left(\frac{1}{(10n+7\alpha)!}\right.\\
&\left.+\frac{((6n+3\alpha)!)}{3((6n+4\alpha)!)}\right)((14n+9\alpha)!)\left(\frac{(\omega t)^{14n+10\alpha}}{(14n+10\alpha)!}\right).
\end{aligned}
\tag{68}
$$

So, we can express the solution as equation (30)

$$
\begin{aligned}
\phi(t) &= \sum_{n=0}^{\infty}\phi_n(t)\\
&= \phi_0(t)+\phi_1(t)+\phi_2(t)+\phi_3(t)+\cdots\\
&= \frac{V}{\omega^{\alpha}}\sum_{n=0}^{\infty}(-1)^n\frac{(\omega t)^{2n+\alpha}}{(2n+\alpha)!}-\frac{\beta\mu}{\omega^{\alpha}}\sum_{n=0}^{\infty}(-1)^n\left(\frac{(\omega t)^{2n+2\alpha}}{(2n+2\alpha)!}\right)\\
&-\frac{\gamma\mu^3}{\omega^{\alpha}}\sum_{n=0}^{\infty}\frac{(-1)^{3n}(6n+3\alpha)!}{((2n+\alpha)!)^3}\left(\frac{(\omega t)^{6n+4\alpha}}{(6n+4\alpha)!}\right)+\frac{\beta^2\mu}{\omega^{2\alpha}}\sum_{n=0}^{\infty}(-1)^n\left(\frac{(\omega t)^{2n+3\alpha}}{(2n+3\alpha)!}\right)\\
&+\frac{\beta\gamma\mu^3}{\omega^{2\alpha}}\sum_{n=0}^{\infty}(-1)^{3n}\left(\frac{((6n+3\alpha)!)}{((2n+\alpha)!)^3}+\frac{3((6n+4\alpha)!)}{((2n+\alpha)!)^2(2n+2\alpha)!}\right)\left(\frac{(\omega t)^{6n+5\alpha}}{(6n+5\alpha)!}\right.\\
&+\frac{3\gamma^2\mu^5}{\omega^{2\alpha}}\sum_{n=0}^{\infty}\frac{(-1)^{5n}((6n+3\alpha)!)((10n+6\alpha)!)}{((2n+\alpha)!)^5((6n+4\alpha)!)}\left(\frac{(\omega t)^{10n+7\alpha}}{(10n+7\alpha)!}\right)\\
&-\frac{\beta^3\mu}{\omega^{3\alpha}}\sum_{n=0}^{\infty}(-1)^n\left(\frac{(\omega t)^{2n+4\alpha}}{(2n+4\alpha)!}\right)\\
&-\frac{\beta^2\gamma\mu^3}{\omega^{3\alpha}}\sum_{n=0}^{\infty}(-1)^{3n}\left(\frac{((6n+3\alpha)!)}{((2n+\alpha)!)^3}+\frac{3((6n+4\alpha)!)}{((2n+\alpha)!)^2((2n+2\alpha)!)}\right.
\end{aligned}
\tag{69}
$$

$$+\frac{3((6n+5\alpha)!)}{((2n+\alpha)!)^2((2n+3\alpha)!)}+\frac{3((6n+5\alpha)!)}{((2n+\alpha)!)(2n+2\alpha)!^2}\Bigg)\left(\frac{(\omega t)^{6n+6\alpha}}{(6n+6\alpha)!}\right)$$
$$-\frac{3\beta\gamma^2\mu^5}{\omega^{3\alpha}}\sum_{n=0}^{\infty}(-1)^{5n}\Bigg(\frac{((6n+3\alpha)!)((10n+6\alpha)!)}{((2n+\alpha)!)^5((6n+4\alpha)!)}$$
$$+\frac{((6n+3\alpha)!)((10n+7\alpha)!)}{((2n+\alpha)!)^2((6n+5\alpha)!)((2n+\alpha)!)^3}$$
$$+\frac{3((6n+4\alpha)!)((10n+7\alpha)!)}{((2n+\alpha)!)^2((6n+5\alpha)!)((2n+\alpha)!)^2((2n+2\alpha)!)}$$
$$+\frac{2((6n+3\alpha)!)((10n+7\alpha)!)}{((2n+\alpha)!)^2((6n+5\alpha)!)((2n+\alpha)!)^2(2n+2\alpha)!}\Bigg)\left(\frac{(\omega t)^{10n+8\alpha}}{(10n+8\alpha)!}\right)$$
$$-\frac{9\gamma^3\mu^7}{\omega^{3\alpha}}\sum_{n=0}^{\infty}\frac{(-1)^{7n}((6n+3\alpha)!)((10n+6\alpha)!)}{((2n+\alpha)!)((6n+4\alpha)!)((2n+\alpha)!)^6}\Bigg(\frac{1}{((10n+7\alpha)!)}$$
$$+\frac{((6n+3\alpha)!)}{3((6n+4\alpha)!)}\Bigg)((14n+9\alpha)!)\left(\frac{(\omega t)^{14n+10\alpha}}{((14n+10\alpha)!)}\right)+\cdots$$

5 Numerical experiments

For the assumed typical values of circuit parameters: $c = 4\times10^{-4}$ farad, $V_0 = 20$ volt, $N = 1000$, $a_1 = 1$, $a_2 = 3$, and the values of constants in (69) are displayed in Table 1. While Figure 1 represents the solution of the fractional nonlinear RLC circuit equation using the FNDM for different α values.

Table 1: The values of constants in Equation (69)

constants	values
$\beta = \frac{a_1}{cN}$	0.25
$\gamma = \frac{a_2}{cN}$	0.75
$V = \frac{\omega V_0}{N}$	1
ω	50
$\mu = \frac{V}{\omega^\alpha}$	$\frac{1}{50^\alpha}$

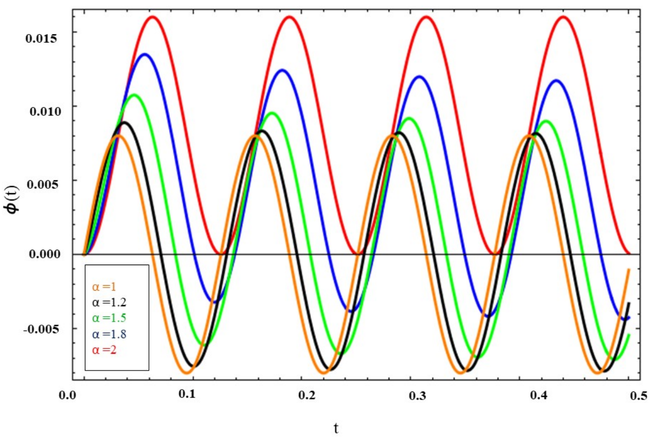

Figure 1: Represents relation between the magnetic flux $\phi(t)$ of the nonlinear RLC circuit investigated by FNDM as a function of time $T(t)$ fractional nonlinear RLC circuit equation using the FNDM for different alpha values, namely $\alpha = 2, \alpha = 1.8, \alpha = 1.5, \alpha = 1.2$, and $\alpha = 1$

We take the approximate solution of the equation (31) in case $\alpha = 2$, the equation (69) becomes

$$\begin{aligned}\phi(t) = &\\ & -(N_1+M_1+G_1)\left(\frac{t^2}{2!}\right)+(M_2+G_2)\left(\frac{t^4}{4!}\right)-G_3\left(\frac{t^6}{6!}\right)\\ & +\left(-(M_3+G_4)(\sin\omega t)+(M_4+G_5)(\sin 2\omega t)-(G_6)(\sin 3\omega t)+(G_7)(\sin 4\omega t)\right)(t)\\ & -\left(-(G_8)(\sin(\omega t))+(G_9)(\sin 2\omega t)\right)(t^3)\\ & -\left(-(M_5+G_{10})(\cos\omega t)+(M_6+G_{11})(\cos 2\omega t)-(G_{12})(\cos 3\omega t)+(G_{13})(\cos 4\omega t)\right)(t^2)\\ & +\left(-(G_{14})(\cos\omega t)+(G_{15})(\cos 2\omega t)\right)(t^4)\\ & +(N_2+M_7+\mu+G_{16})(1-\cos\omega t)\\ & -(N_3+M_8+G_{17})(1-\cos 2\omega t)\\ & +((N_4)+M_9+G_{18})(1-\cos 3\omega t)\\ & -(M_{10}+G_{19})(1-\cos 4\omega t)\\ & +(M_{11}+c_{20})(1-\cos 5\omega t)\\ & -G_{21}(1-\cos 6\omega t)+G_{22}(1-\cos 7\omega t)+\cdots\end{aligned} \tag{70}$$

And we get the same general formula for the solution that we obtained with the HPTM method, Which we can write as

$$\begin{aligned}\phi(t) = & \sum_{n=1}^{\infty} A_{(n)}(-1)^n\left(\frac{t^{2n}}{2n!}\right)+\sum_{n=1}^{\infty}(-1)^n\left[t^{2n}\left(\sum_{j}^{\infty}(-1)^j B_n^j\cos j\omega t\right)\right.\\ & \left.-t^{2n-1}\left(\sum_{k=1}^{\infty}(-1)^k C_n^k\sin k\omega t\right)\right]\\ & -\sum_{n=1}^{\infty}(-1)^n D_n(1-\cos n\omega t).\end{aligned} \tag{71}$$

Figure 2 shows the congruence of the solution with the previous methods HPTM (Thunibat et al., 2021) and the FNDM method when imposing the values of the circuit parameters numerically: $c = 4\times 10^{-4}$ farad, $V_0 = 20$ volt, $N = 1000$, $a_1 = 1$, $a_2 = 3$.

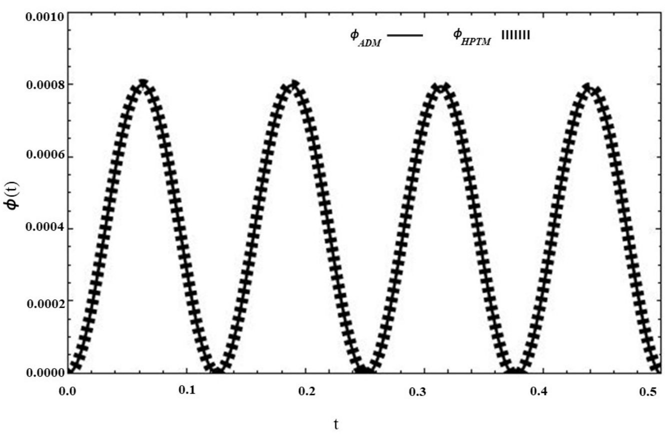

Figure 2: Represents the relation between the magnetic flux $\phi(t)$ of the nonlinear RLC circuit investigated by FNDM, and HPTM as a function of time $T(t)$

6 Conclusions

In this work, we represent a particular solution for the fractional nonlinear RLC circuit equation. The solution is expressed using Fractional FNDM with Caputo definition. This technique displayed as a powerful tool that allows us to handle a variety of differential equations of nonlinear fractional order. Assuming some numerical examples for different values of α, the behavior and correctness of the solution determined by the current method is confirmed graphically. These graphs prove a simple harmonic oscillation behavior displayed as a function of magnetic flux and time. The FNDM model affixes different values of α to display for fragment from 1 to 2 to display the proposed inductance and capacitor that predict the significant effect of the fragments as a control button for amplitude and frequency on the magnetic flux and therefore the electric current. It is clearly obvious that the magnetic field is shifting in the positive direction presenting the alternative magnetic flux superimposed on a constant magnetic field. Otherwise, the increase in α provides an increasing in the constant magnetic field. On the other hand, we discovered a particular agreement between the solutions provided by the method of HPTM when $\alpha = 2$ with the solution of our current method FNDM. We illustrated this agreement with a numerical example and by graphically representing the solutions provided through the methods. Lastly, this research provides a model that success to prove the graphical explanation, discussing the justification factor, and displaying the harmonic oscillation for the nonlinear RLC with nonlinear inductance using CF definition. But this model is not limited for solving nonlinear inductance only, but it can be improved and spread to include nonlinear capacitance and nonlinear resistance to predict quantitative and qualitative physical properties as current and magnetic field. . . etc.

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